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Evaluation of nonlinear models for time-based rates demand response programs

H.A. Aalami [a,*], M. Parsa Moghaddam [b], G.R. Yousefi [c]

[a] Department of Electrical Engineering, Imam Hussein University, Tehran, Iran
[b] Faculty of Electrical & Computer Engineering, Tarbiat Modares University, Tehran, Iran
[c] Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, Iran

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ABSTRACT

Demand Response (DR) programs have been implemented in many competitive electricity markets to prevent price spikes and power systems unreliability. Mathematical modeling of these programs helps regulators to evaluate the impact of price responsive loads on market conditions. In this paper, several nonlinear economic models of price responsive loads are derived based on price elasticity of demand and customer benefit function. The main objectives of the paper include extracting different mathematical models for Time of Use (TOU) programs, and comparing these models to find out which model shows more conservative and which one shows non-conservative results compared with the initial load curve. This could be used by ISOs or DR programs developers as a guideline to use conservative models to have lower error in load profile characteristics estimation, such as variation in peak load or amount of energy consumptions. In order to evaluate the performance of the proposed nonlinear DR models, numerical studies are conducted on the load curve of different markets. Results obtained by using different models are presented and compared considering different scenarios for price, elasticity and potentiality of DR programs implementation. Characteristics of both linear and nonlinear economic models of price responsive loads have been evaluated.

Introduction

Over the last two decades, electricity markets have been involved in restructuring aimed at promoting competition among market participants. One typical market design often requires electricity to be sold in a spot wholesale market where potential buyers (retailers or end consumers) and sellers (producers) submit their bids for each time period. The demand and the supply sides meet in the electricity power exchanges. The resulting auction yields the equilibrium prices, which vary over time and space, considering network constraints and supply/demand balance. However, a common feature of the electricity wholesale markets is lack of price responsiveness measured by the value of demand elasticity [1]. This is due not only to the peculiar characteristics of the commodity, such as non-storability, lack of good substitutes, and the relatively small impact of electricity bill on the typical consumer's budget, but also to the relation between wholesale and retail markets. Since end users simply do not see the "true" spot prices, they cannot use these prices when making decisions regarding power withdrawal; this "inelastic" behavior is transmitted to retailers, who have legal obligations to serve their customers and therefore to the wholesale demand. Furthermore, the overwhelming lack of interest from consumers in seeing the real price of electricity makes it politically difficult to implement demand elasticity improvement measures [2].

In these circumstances, Demand Response (DR) programs are such useful tools for the independent system operator (ISO) that can be implemented at times of critical system conditions to provide the much needed system demand reduction and an operating reserve that can be activated within a relatively short time. The idea is to make it attractive for customers to use less power during periods of peak load [3]. In a DR program, the customer signs a contract with the retailer, local utility or the ISO to reduce its demand as and when requested. The utility benefits from reduction of its peak load and thereby saving costly generation reserves, restoring quality of service and ensuring reliability [4,5]. The customer benefits from reduction in its various energy levels, costs and particularly from incentives provided by the local utility or the ISO. Utilities typically commit their expected energy requirements with a mix of bilateral forward contracts with generators and purchases

* Corresponding author. Tel.: +98 21 77104901; fax: +98 21 77104907.
E-mail addresses: halami@ihu.ac.ir (H.A. Aalami), parsa@modares.ac.ir (M. Parsa Moghaddam), yousefi@cc.iut.ac.ir (G.R. Yousefi).

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Nomenclature

i	For i hour	E	Price elasticity of demand
j	For j hour	$E(i,i)$	Self-elasticity
$B_0\,(i)$	Customer's benefit when the demand is equal to The nominal value $d_0(i)$ ($)	$E(i,j)$	Cross-elasticity
$B(d(i))$	Customer's benefit in i-th hour ($)	ρ_0	Initial electricity price ($/kW)
d_0	Initial demand value (kW)	$\rho_0(i)$	Initial electricity price in i-th hour ($/kW h)
$d_0(i)$	Initial demand value in i-th hour (kW h)	ρ	Spot electricity price ($/kW)
d	Customer demand (kW)	$\rho(i)$	Spot electricity price in i-th hour ($/kW h)
$d(i)$	Customer demand in i-th hour (kW h)	S	Customer's profit ($)

in day-ahead and real-time markets. The extent of customer savings from price reductions thus depends on how much energy is purchased in spot markets [6].

The programs are usually structured into one of two categories: Incentive-Based Programs (IBP) and Time-Based rate Programs (TBR). Each of these categories is composed of several programs as indicated in Fig. 1. In time-based rate programs (Time of Use (TOU), Real Time Pricing (RTP), Critical Peak Pricing (CPP)) the electricity price changes for different periods. Incentive-based programs include Direct Load Control (DLC), Emergency Demand Response Program (EDRP), Interruptible/Curtailable service (I/C), Capacity market Program (CAP), Demand Bidding (DB) and Ancillary Service (A/S) programs. More detailed explanations of DR programs can be found in [7–10]. In this paper, we have focused on TOU program which is briefly introduced in the following.

In TOU program, the electricity price changes over different periods according to the electricity supply cost. For example, high price for peak period, medium price for off-peak and low-price for low load period, and there isn't any incentive or penalty for this program. Definition of TOU periods differs widely among utilities based on the timing of their peak system demands over the day, week, or year [3]. In order to evaluate the impact of DR programs on the network and market characteristics such as load profile, transmission congestion and reserve margin , developing price responsive demand models is necessary. Obviously, there are many possible structural forms for customer response. Linear economic models of price responsive loads for DR programs have been developed in [1,11–16]. Since the optimization problem of the customer profit is nonlinear, it is necessary to develop nonlinear economic models of price responsive loads for more realistic characterization of the demand. Maximization of the utility benefit function problem by using different nonlinear benefit-demand functions has

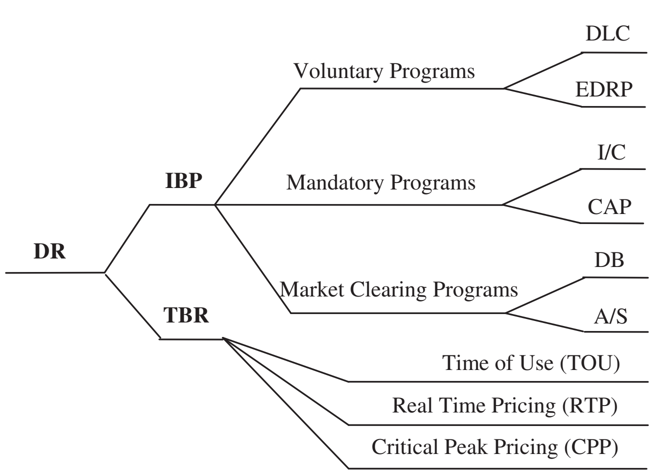

Fig. 1. Categories of demand response programs.

been discussed in [17,18]. In [19], a retailer profit is maximized through using a nonlinear load model with power structure.

In this paper, three nonlinear structures namely; power, exponential and logarithmic economic models of price responsive loads for DR programs are extracted by using the concept of "price elasticity of demand", and "customer benefit function". These models are compared with linear ones to determine the accuracy and consistency with operational strategies. The proposed models can be used to analyze the impact of DR programs on load profile characteristics.

The main contribution of this paper includes extracting different mathematical models for TOU programs, and comparing these models to find out which model shows more conservative and which one shows non-conservative results compared with the initial load curve. This could be used by ISOs or DR programs developers as a guideline to use conservative models to have lower error in load profile characteristics estimation, such as variation in peak load or the amount of energy consumptions. Therefore, they would have a better and more realistic insight in power systems and market operation and in performing related tasks, such as reserve procurement. Furthermore, as another contribution, a procedure is introduced for the selection of the more reliable load economic model for analyzing the impact of implementation of DR programs on power system characteristics. It should be noted that linear model of demand response programs have been discussed in previous works of the authors Refs. [11–13] but, in this study various nonlinear models are developed. Furthermore, in this manuscript the results of the developed nonlinear models have been compared with linear model of previous works.

The remainder of the paper is organized as follows: In Section 'Nonlinear modeling of DR programs', nonlinear models of DR programs are derived. Section 'Numerical studies' is devoted for numerical studies considering different scenarios for price, elasticity and program potentiality for evaluation of proposed nonlinear models of DR programs. Furthermore, sensitivity analysis of models to change price, elasticity and program potentiality has been done by standard deviation. Finally, Section 'Conclusions' concludes the paper.

Nonlinear modeling of DR programs

In order to model the customer response, we consider customer demand for electricity $d(i)$ and assume that it depends on price or tariff that consumer must pay for electricity, $\rho(i)$. Obviously, there are many possible structural forms for customer response. In this paper three such structures namely; power, exponential and logarithmic nonlinear models for customer response are derived. It is important to note that a second-order Taylor Series expansion of the power, exponential and logarithmic benefit functions $B(d(i))$ about $d(i) = d_0(i)$ yields the quadratic income function, and a first-order Taylor Series expansion of the response function $d(i)$ resulting from the power, exponential and logarithmic functions yields the linear response functions obtained from the quadratic income function. Hence, if $d(i) \cong d_0(i)$, all three structures are equivalent. However, in actual applications, the assumption $d(i) \cong d_0(i)$, may not be valid, in which case a choice of explicit structures is required but, the best model is not known [20].

In the following, the above mentioned three nonlinear structures for customer response are derived for single period, multiple period and composite period loads which include both single and multiple period loads.

Power modeling

Modeling single period elastic loads

Elasticity is defined as the demand sensitivity with respect to the price [21]:

$$E(i,j) = \frac{\partial d(i)}{\partial \rho(j)} \cdot \frac{\rho_0(j)}{d_0(i)} \quad j = 1, 2, 3, \ldots, 24 \tag{1}$$

$$\begin{cases} E(i,j) \le 0 & \text{if } i = j \\ E(i,j) \ge 0 & \text{if } i \ne j \end{cases} \tag{2}$$

The elasticity coefficient in (1) indicates the relative change in demand for the i-th hour that would be resulted from a change in the electricity price in the j-th hour. The demand of the i-th hour decreases as the price of this hour increases and it will increase as the price of the j-th hour increases. The term $\rho_0(j)/d_0(i)$ in (1) is used for normalization. If the electric energy prices vary for different periods, then the demand reacts one of the followings:

- Some of loads are not able to move from one period to another (e.g. illuminating loads) and they could be only "on" or "off". So, such loads have a sensitivity just in a single period and it is called "self elasticity", and always has a negative value, $E(i,i)$.
- Some consumption could be transferred from the peak period to the off-peak or low periods. Such behavior is called multi period sensitivity and is evaluated by "cross elasticity". This value is always positive, $E(i,j)$.
- Some large loads (e.g. industrial loads/regional electricity networks and electricity markets) may include both single and multi-period loads which are referred to here as composite period loads.

Let $B(d(i))$ be the benefit of customer during i-th hour obtained due to consumption of $d(i)$ kWh of electric energy. Then, the customer's profit, S, for the same period will be as follows:

$$S = B(d(i)) - d(i) \cdot \rho \tag{3}$$

According to the classical optimization rules, the maximum profit of the customer can be calculated as:

$$\frac{\partial S}{\partial d(i)} = \frac{\partial B(d(i))}{\partial d(i)} - \rho(i) = 0 \tag{4}$$

$$\frac{\partial B(d(i))}{\partial d(i)} = \rho(i) \tag{5}$$

For power structure of customer response function, the benefit function can be obtained by Taylor expansion of $B(d(i))$ as following [18]:

$$B(i) \cong B_0(i) + \frac{\rho_0(i)d(i)}{1 + E(i,i)^{-1}} \left\{ \left(\frac{d(i)}{d_0(i)} \right)^{E(i,i)^{-1}} - 1 \right\} \tag{6}$$

where the self elasticity, $E(i,i)$, is defined as the variation of load in the i-th period due to the electricity price change in the same period. The function is concave and non-decreasing and is not defined for a zero load value or for elasticity equal to -1; also negative values for the benefit may be obtained. The marginal benefit in power model is greater than this value in the linear case and the benefit is reduced for low values of the elasticity. By differentiating the above equation with respect to $d(i)$,we will have:

$$\frac{\partial B(d(i))}{\partial d(i)} = \frac{\rho_0(i)}{1 + E(i,i)^{-1}} \left\{ \left(\frac{d(i)}{d_0(i)} \right)^{E(i,i)^{-1}} - 1 \right\} + \frac{\rho_0(i).d(i)}{1 + E(i,i)^{-1}} \left\{ E(i,i)^{-1} \cdot \frac{1}{d_0(i)} \left(\frac{d(i)}{d_0(i)} \right)^{E(i,i)^{-1}-1} \right\} \tag{7}$$

Substituting (5) in (7) results in:

$$(1 + E(i,i)^{-1}) \cdot \frac{\rho(i)}{\rho_0(i)} = \left(\frac{d(i)}{d_0(i)} \right)^{E(i,i)^{-1}} - 1 + E(i,i)^{-1} \cdot \left(\frac{d(i)}{d_0(i)} \right)^{E(i,i)^{-1}} \tag{8}$$

$$\frac{\rho(i)}{\rho_0(i)} = \left(\frac{d(i)}{d_0(i)} \right)^{E(i,i)^{-1}} - \left(\frac{1}{1 + E(i,i)^{-1}} \right) \tag{9}$$

For small values of elasticity, the second term of the above equation can be ignored. It should be notified that in real situations the elasticity is in the range of -0.1, -0.2. Therefore, customer's demand can be represented as following:

$$d(i) = d_0(i) \cdot \left(\frac{\rho(i)}{\rho_0(i)} \right)^{E(i,i)} \tag{10}$$

The above equation represents single period elastic load model.

Modeling multi period elastic loads

To model multi period elastic load first, the concept of cross elasticity should be addressed. The cross elasticity is defined as the variation of load in the i-th period due to the electricity price change in the j-th period, as represented by (1) and (2). For $j \ne i$, the same process having been implemented as the single period model (Eqs. (3)–(10)), following equation can be obtained.

$$d(i) = d_0(i) \cdot \left(\frac{\rho(j)}{\rho_0(j)} \right)^{E(i,j)} \tag{11}$$

Now, by assumption of i = constant, $j \ne i$ and j = 1, 2, 3,. . ., 24, the multi period elastic load model for power structure of customer response function can be obtained as follows:

$$d(i) = d_0(i) \cdot \prod_{\substack{j=1 \\ j \ne i}}^{24} \left(\frac{\rho(j)}{\rho_0(j)} \right)^{E(i,j)} \tag{12}$$

It should be noted that a 24-h interval has been considered in (12). However, longer or shorter intervals are also definable.

Modeling composite period elastic loads

As it was noted earlier, certain demands may include both single and multi-period loads which are referred to here as composite period loads. For power structure of customer response function, for i = constant, and j = 1, 2, 3,. . ., 24 (including i), the composite period load model can be obtained by combining (10) and (12) as follows:

$$d(i) = d_0(i) \cdot \prod_{j=1}^{24} \left(\frac{\rho(j)}{\rho_0(j)} \right)^{E(i,j)} \tag{13}$$

Eq. (13) shows how much the customer's consumption should be to achieve maximum benefit in a 24-h interval while participating in TBR programs. For the power structure, $E(i,i)$ is constant for all $\rho(i)$ and $d(i)$. Hence, the power structure is often called the "constant elasticity" model.

Exponential modeling

Modeling single period elastic loads

For exponential structure of customer response function, the benefit function can be obtained by Taylor expansion of $B(d(i))$ as following [15]:

$$B(i) \cong B_0(i) + \rho_0(i)d(i) \cdot \left\{1 + \frac{1}{E(i,i)}\left[\ln\left(\frac{d(i)}{d_0(i)}\right) - 1\right]\right\} \tag{14}$$

It should be noted that the function is not defined for zero loads as well as zero elasticity. Differentiating the above equation yields:

$$\frac{\partial B(d(i))}{\partial d(i)} = \rho_0(i)\left\{1 + \frac{1}{E(i,i)}\left[\ln\left(\frac{d(i)}{d_0(i)}\right)\right] - 1\right\} + \rho_0(i) \cdot d(i)\left\{\frac{1}{E(i,i)} \cdot \frac{1}{d_0(i)} \cdot \frac{d_0(i)}{d(i)}\right\} \tag{15}$$

Substituting (5) in (15) results in:

$$\rho(i) = \rho_0(i) + \frac{\rho_0(i)}{E(i,i)}\left[\ln\left(\frac{d(i)}{d_0(i)}\right) - 1\right] + \frac{\rho_0(i)}{E(i,i)} \tag{16}$$

$$\rho(i) - \rho_0(i) = \frac{\rho_0(i)}{E(i,i)}\left[\ln\left(\frac{d(i)}{d_0(i)}\right) - 1 + 1\right] \tag{17}$$

Therefore, customer's demand can be represented as follows:

$$d(i) = d_0(i) \cdot \mathrm{EXP}\left\{E(i,i)\frac{\rho(i) - \rho_0(i)}{\rho_0(i)}\right\} \tag{18}$$

The above equation represents single period elastic load model.

Modeling multi period elastic loads

Using the cross elasticity definition (1) and (2), by assuming i = constant, $j \neq i$ and $j = 1, 2, 3, \ldots, 24$, similar Section 'Power modeling'), we see the multi period elastic load model for exponential structure of customer response function can be obtained as follows:

$$d(i) = d_0(i) \cdot \mathrm{EXP}\left\{\sum_{\substack{j=1\\ j\neq i}}^{24} E(i,j)\frac{\rho(j) - \rho_0(j)}{\rho_0(j)}\right\} \tag{19}$$

Modeling composite period elastic loads

For exponential structure of customer response function, for i = constant, and $j = 1, 2, 3, \ldots, 24$ (including i), the composite period load model can be obtained by combining (18) and (19) as follows:

$$d(i) = d_0(i) \cdot \mathrm{EXP}\left\{\sum_{j=1}^{24} E(i,j)\frac{\rho(j) - \rho_0(j)}{\rho_0(j)}\right\} \tag{20}$$

Logarithmic modeling

Modeling single period elastic loads

For logarithmic structure of customer response function, the benefit function can be obtained by Taylor expansion of $B(d(i))$ as following [15]:

$$B(i) \cong B_0(i) + \rho_0(i)d_0(i)E(i,i) \cdot \left\{\mathrm{EXP}\left[\left(\frac{d(i) - d_0(i)}{E(i,i) \cdot d_0(i)}\right) - 1\right]\right\} \tag{21}$$

Differentiating the above equation yields:

$$\frac{\partial B(d(i))}{\partial d(i)} = \rho_0(i)d_0(i)E(i,i) \cdot \left(\frac{1}{E(i,i) \cdot d_0(i)}\right) \cdot \mathrm{EXP}\left(\frac{d(i) - d_0(i)}{E(i,i) \cdot d_0(i)}\right) \tag{22}$$

Substituting (5) in (22) results in:

$$\rho(i) = \rho_0(i) \cdot \mathrm{EXP}\left(\frac{d(i) - d_0(i)}{E(i,i) \cdot d_0(i)}\right) \tag{23}$$

$$\frac{d(i) - d_0(i)}{E(i,i) \cdot d_0(i)} = \ln\left(\frac{\rho(i)}{\rho_0(i)}\right) \tag{24}$$

Therefore, customer's demand can be represented as following:

$$d(i) = d_0(i) \cdot \left\{1 + E(i,i) \cdot \ln\left(\frac{\rho(i)}{\rho_0(i)}\right)\right\} \tag{25}$$

The above equation represents single period elastic load model.

Modeling multi period elastic loads

Using the cross elasticity definition of (1) and (2), by assuming i = constant, $j \neq i$ and $j = 1, 2, 3, \ldots, 24$, similar Section 'Power modeling', we see the multi period elastic load model for logarithmic structure of customer response function can be obtained as follows:

$$d(i) = d_0(i) \cdot \left\{1 + \sum_{\substack{j=1\\ j\neq i}}^{24} E(i,j)\ln\left(\frac{\rho(j)}{\rho_0(j)}\right)\right\} \tag{26}$$

Modeling composite period elastic loads

For logarithmic structure of customer response function, for i = constant, and $j = 1, 2, 3, \ldots, 24$ (including i), the composite period load model can be obtained by combining (25) and (26) as follows:

$$d(i) = d_0(i) \cdot \left\{1 + \sum_{j=1}^{24} E(i,j)\ln\left(\frac{\rho(j)}{\rho_0(j)}\right)\right\} \tag{27}$$

Figs. 2 and 3, illustrate the behaviors of nonlinear models of demand response versus linear one for different values of elasticity and different ratios of spot electricity price to initial price (ρ/ρ_0), respectively. As seen in Fig. 2, the higher the elasticity, the more divergence there is between the responses of the models. This figure shows that for small elasticity, where the demand does not change too much from its initial value, all model structures behave almost the same. Furthermore, it can be seen that the response of linear model for high elasticity values does not match with actual situation.

From Fig. 3, it can be observed how different demand response functions have similar performance for the range of lower ratios of spot electricity price to initial price, where they can be approximated by a linear demand function. However, there are considerable differences between various models for higher price ratios. This result recommends that in case of price spikes, nonlinear model with power structure is the most conservative one.

Numerical studies

In this case study, TOU program, as one of the TBR programs, is used for evaluation of the proposed nonlinear price responsive load models. In this regard, different patterns of load profiles of

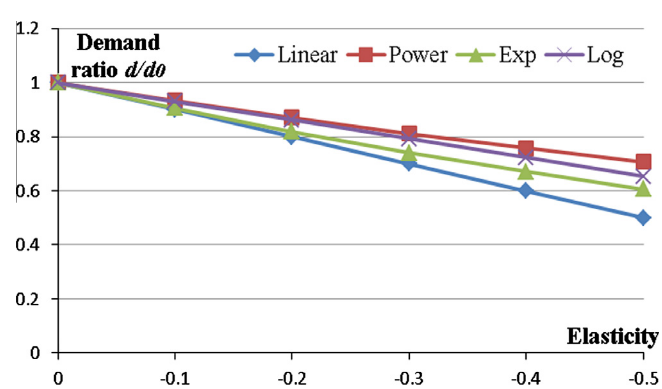

Fig. 2. Demand response functions for different elasticities.

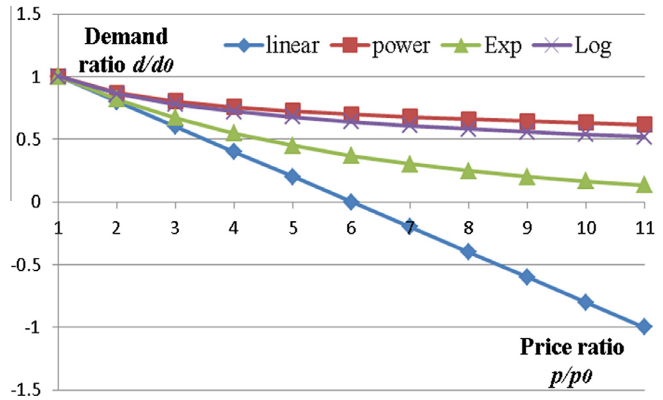

Fig. 3. Demand response functions for different ratios of spot electricity price to initial price ($E(i,i) = -0.2$).

different real world and test networks such as the Iranian Power Grid in 2008 [22], a province of China [23], the IEEE-RTS test system [24] and, PJM market [25] are used for our numerical studies. Initial load curves are presented in Fig. 4. In a three -tariff system, these curves are divided into three different periods: valley, off-peak load and peak load. The average price of electricity has been considered 0.04, 0.016 and 0.004 $/kW h, in peak, off-peak and valley periods, respectively [22]. The price elasticity of the demand are considered as listed in Table 1, which is originally taken from [26]. Although different customers (residential, commercial and Industrial) have various amounts of demand-price elasticity, and it is possible to extract more complicated models considering different elasticity values, there will not be significant changes in the results. Definitely, the results will be more accurate, but there is not any change in a conservative modeling.

Several scenarios have been considered as indicated in Table 2. These scenarios are designed based on the changes of elasticity, electricity price and potential of TOU program. Here, the potentiality of program means the expected capacity of customer participation in the program. The results of the simulation studies and the impact of the proposed nonlinear models as well as the linear model of price responsive loads on load curve characteristics are discussed for different scenarios as follows.

Scenario 1 In this scenario, TOU pricing is 0.04, 0.016 and 0.004 $/kW h, in peak, off-peak and valley periods, respectively. According to the investigations of mentioned networks, it is assumed that the potential of implementation of TOU program is

Table 1
Self and cross elasticity values.

	Peak	Off-peak	Valley
Peak	−0.20	0.016	0.012
Off-peak	0.008	−0.20	0.01
Valley	0.006	0.008	−0.20

15%. This scenario is considered as the base case for comparison with other scenarios. By applying composite models of linear, power, exponential and logarithm structures on the initial load curves, the results of all models are almost similar which can be seen in Fig. 5. As it was expected, since the elasticities and price ratios are low, different models have very similar performance and they can be approximated reasonably by the linear model. More precise analysis of the results of the above figure shows a maximum 2% difference between the linear and power model during peak period. For certain power system problems such as assigning reasonable reserve margin for reliable operation of the system, the above error may be considerable. As it is shown in Fig. 5, for Iranian peak day of 2008, the amount of peak in initial load at 22:00 is 39,696 MW. When TOU program is implemented using different linear and nonlinear models, the resulted peak value will be 34,906 MW, 35,465 MW, 35,246 MW, and 35,215 MW for linear, Power, exponential, and logarithmic models, respectively. It can be seen that the linear model shows the most difference and the power model shows the least difference with the initial peak load. Therefore, more conservative model i.e. power structure should be selected. The behavior of different models regarding the energy consumption criterion can be obtained from Fig. 5.

Scenario 2 The impact of increasing elasticity on the models behavior has been evaluated in this scenario. Fig. 6 shows the response of different models to TOU program when the elasticities are doubled in comparison with scenario 1. As it is shown in Fig. 6, for Iranian peak day of 2008, the amount of peak in initial load at 22:00 is 39,696 MW. When TOU program is implemented using different linear and nonlinear models, the resulted peak value will be 32,843 MW, 34,369 MW, 34,059 MW, and 33,461 MW for linear, Power, exponential, and logarithmic models, respectively. It can be seen that the linear model shows the most difference and the power model shows the least difference with the initial peak load. In addition, the resulted load curves are more dispersed than those of scenario 1. There exists a maximum 6% difference on the peak point of the load curves. This is in consistence with the results of Fig. 2.

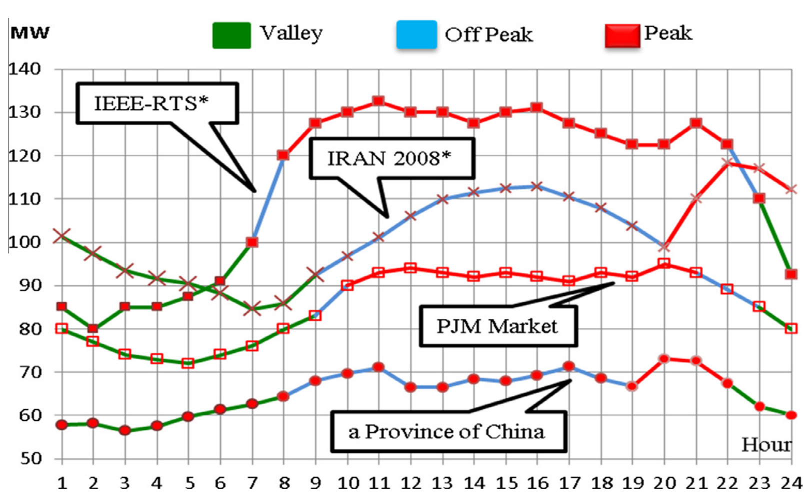

Fig. 4. Different networks initial load curves. *IEEE-RTS multiplied by 0.05 and Iran 2008 multiplied by 0.0032 for more clarification.

Table 2
Definition of scenarios.

Scenario no	Program	Model	Elasticity	Electricity price	Potential (%)
1	TOU (base case)	Linear Power Exponential Logarithm	As Table 1	0.04, 0.016 and 0.004 $/kW h, in peak, off-peak and valley periods	15
2	TOU (double elasticity)	Linear Power Exponential Logarithm	Double values of scenario 1	As scenario 1	As scenario 1
3	TOU (double price)	Linear Power Exponential Logarithm	As scenario 1	0.08, 0.016 and 0.002 $/kW h, in peak, off-peak and valley periods	As scenario 1
4	TOU (double potential)	Linear Power Exponential Logarithm	As scenario 1	As scenario 1	Double value of scenario 1

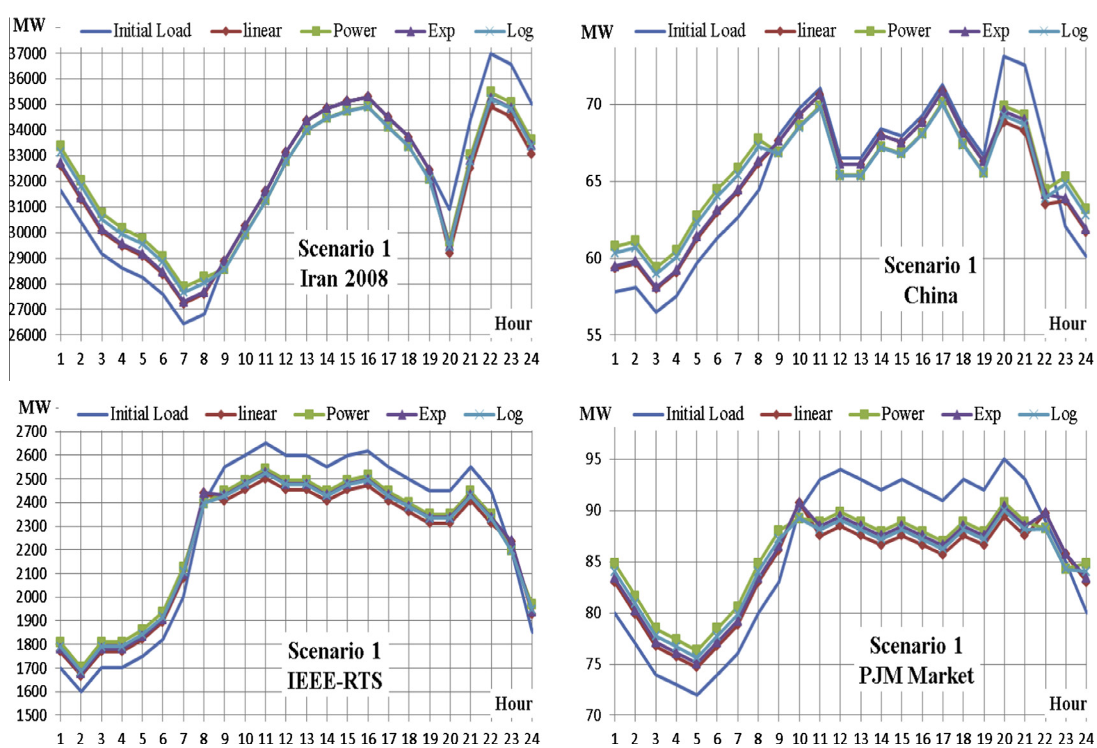

Fig. 5. Different models for TOU program of scenario 1 (base case).

Scenario 3 In order to investigate the effect of the change of electricity prices on the behavior of different models, the prices are doubled in comparison with scenario 1. The results of this case are presented in Fig. 7. It can be seen that the obtained load profiles for different models obey the results of Fig. 5. In other words, for higher electricity prices the nonlinear behavior of the demand response is dominant. In addition, the resulted load curves are more dispersed than other scenarios.

Scenario 4 In this scenario the potential of implementation of TOU program is raised to 30%. In scenario 4, and other scenarios, the program potential is assumed to be as a percentage of $d_0(i)$ that acts in the models and the obtained result is added to the remaining part of $d_0(i)$.The results of this scenario are depicted in Fig. 8. Dispersion of load curves can be observed in peak period in comparison with scenario 1. A 4% difference between linear and power model can be seen on peak points of the resulted load profiles. The impacts of different models of TOU program on different power systems peak and total energy consumption for different scenarios are illustrated in Figs. 5–8. In scenarios 2, 3 and 4, all of the models show more reduction in peak load and energy consumption, when the price/elasticity/program potential increases, compared with scenario 1.

Furthermore, from Figs. 5–8 it can be seen that in both high and low load conditions the power model is the most conservative model in all scenarios while the linear model is the most non conservative one. These results show that the performance of other structures i.e. exponential and log, are located between the performances of the above mentioned models. The above results show that if the ISO plans to implement TOU program based on the linear model, the operational targets may not be achieved and the power

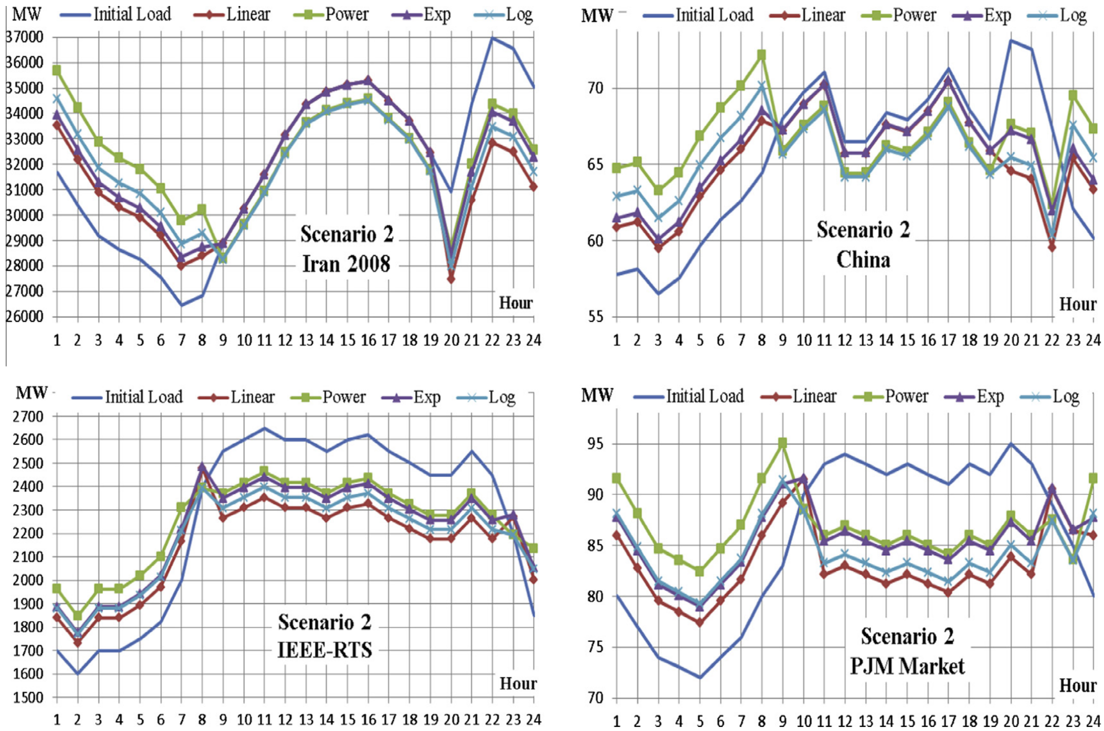

Fig. 6. Different models for TOU program of scenario 2 (double elasticity).

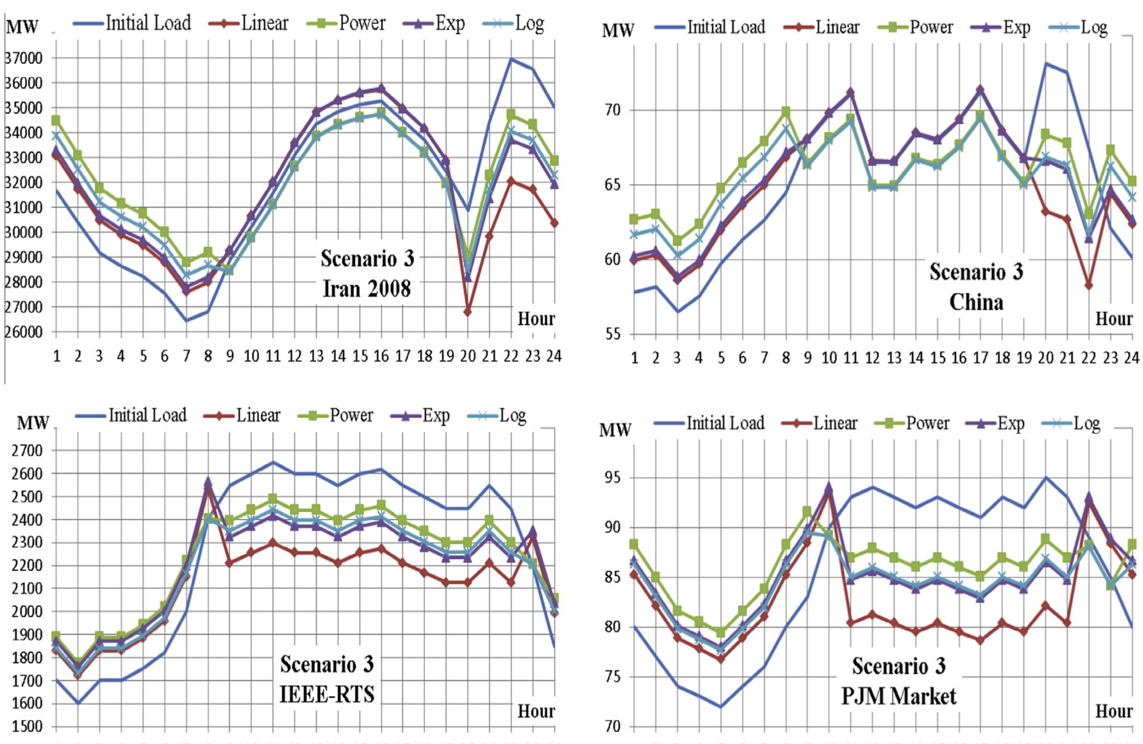

Fig. 7. Different models for TOU program of scenario 3 (double price).

systems reliability may be jeopardized. Obviously, to achieve a high reserve security margin through peak reduction conservative nonlinear models are recommended. Another considerable point that can be seen in Figs. 5–8 is that the higher values of elasticity, price and program potential will lead to the higher divergence between models. Table 3 shows the standard deviation (*SD*) of different models in each scenario which is calculated using (28).

$$SD = \sqrt{\frac{\sum (x - \bar{x})^2}{(n - 1)}} \tag{28}$$

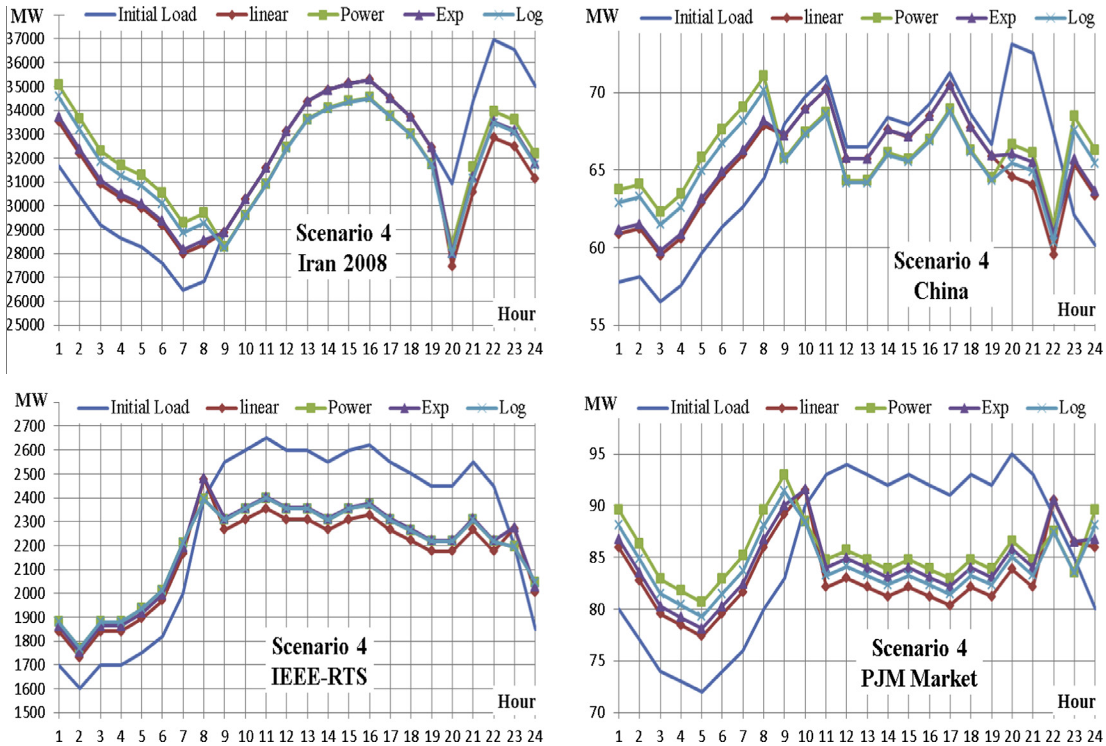

Fig. 8. Different models for TOU program of scenario 4 (double potential).

Table 3
Standard deviation of different models.

Network	Scenario no	Scenario description	Standard deviation	
			Peak at peak period* (MW)	Total energy consumption (MWh)
IRAN 2008	1	Base case	230	1814
	2	Double elasticity	674	6783
	3	Double price	1130	5468
	4	Double potential	460	3628
The province of CHINA	1	Base case	0.4	4.1
	2	Double elasticity	1.4	15
	3	Double price	2.2	10.3
	4	Double potential	0.9	8.2
IEEE-RTS	1	Base case	16.5	317
	2	Double elasticity	48.3	1007
	3	Double price	81	1173
	4	Double potential	23	398
PJM market	1	Base case	0.6	11
	2	Double elasticity	1.9	37
	3	Double price	2.8	36.7
	4	Double potential	1.2	22.7

* The peak time at peak period for Iran 2008 was happened at 22:00, as well as, the province of China at 20:00, IEEE-RTS at 11:00 and PJM market at 20:00 h.

where x is the sample, $\bar{x}$ is the mean average and n is the number of samples. Standard deviation for peak at peak period has been calculated from numerical values of peak for four models for each network as well as total energy consumption. For example at 22:00 PM (the Iran 2008 peak) the value of peak has been obtained by each DR model. Standard deviation has been calculated from these four values. It can be seen that high standard deviations are obtained for scenarios 2, 3, 4 and the least one is for scenario 1.

In other words when amounts of energy prices, elasticity and potential of the programs are intermediate, the performances of the models are almost similar i.e. less standard deviation. However, when each of the values of electricity price, elasticity and programs potential increases, the results of the models will be significantly different (high *SD*). For example, Table 3 shows that when the electricity price is doubled, the standard deviation has become about 5 and 3 times more for peak and energy consumption criteria, respectively. According to the above discussions, the degree of reliance of different model structures (more conservative model) can be represented as Fig. 9. Fig. 9 can be used by the ISO as a guideline for selection of relevant models for different system operation strategies i.e. the peak or energy consumption reduction. The above numerical results confirm that the power model has the greatest degree of reliance in comparison with other models and at the opposite side; the linear model has the lowest degree of reliance. Therefore, it can be concluded that applying the power model for investigating the impact of TBR DR Programs on the network

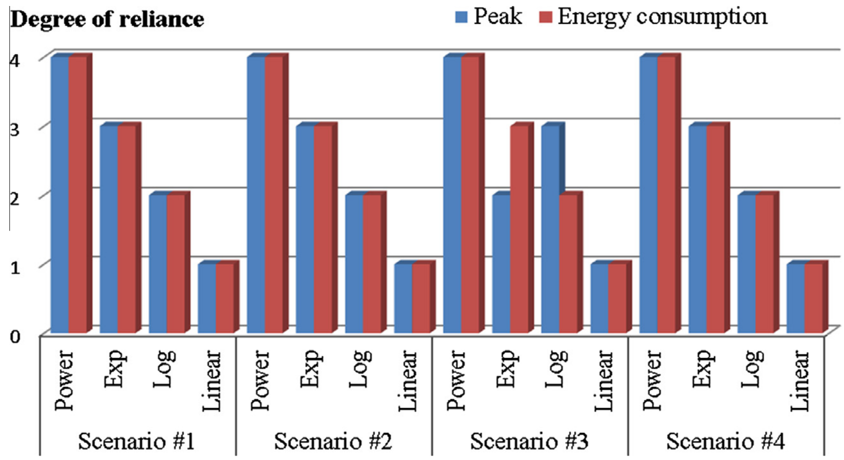

Fig. 9. Degree of reliance of different model structures for all four networks.

operational features could lead to more realistic results. If ISO can obtain the demand response function of the grid such as linear, power, exponential or log, he/she should simulate DR programs with relative model but, if the behavior of the demand is unknown for the ISO, he/she can conduct analysis of the system operation by using the results of Fig. 9. In other words, the higher the degree of reliance, the better the performance of DR models.

Conclusions

Development of nonlinear models for time-based demand response programs has been addressed in this paper. These nonlinear models which are based on the concept of the price elasticity of the demand have different functions including power, exponential and logarithmic structures. The developed models can be used by ISOs or DR programs developers as a guideline to use conservative models to have lower estimation error in load profile characteristics, such as variation in peak load or the amount of energy consumptions. Therefore, they will have a better and more realistic insight in power systems and market operation as well as performing related tasks, such as reserve procurement. Furthermore, a procedure was introduced for the selection of more reliable load economic model for analyzing the impact of implementation of DR programs on power system characteristics.

The behavior of the proposed nonlinear models versus previously developed linear model have been investigated against increasing the elasticity, electricity price and the program potential in different power systems load curves. It has been shown that for small elasticity values as well as small price deviations both the linear and nonlinear models have almost similar behavior. The results of sensitivity analysis clarified that as the elasticity and/or electricity price increase, the differences between the behaviors of the models become more. It was illustrated that the nonlinear model with power structure has the most conservative behavior with high degree of reliance suitable for system operational applications. The developed models have been applied to the load profile curves of actual systems with justifiable results. These models can be used by the ISO to solve certain market issues such as providing required reserve margins on the presence of DR programs.

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